

Albert Cohen's 60th Birthday

Super Resolution Phase Retrieval for Sparse Signals

Gilles Baechler
Miranda Krekovic
Juri Ranieri
Amina Chebira
Yue M. Lu
Martin Vetterli



OVERVIEW

1. **Albert Cohen**
2. Our 2019 paper revisited
3. Super Resolution Phase Retrieval for Sparse Signals
4. Where to go from here
5. Conclusion

UNIVERSITE PARIS IX DAUPHINE - U.E.R. MATHEMATIQUE DE LA DECISION

THESE

POUR L'OBTENTION DU TITRE DE
DOCTEUR
EN ANALYSE NON LINEAIRE APPLIQUEE

Sujet.

*ONDELETTES, ANALYSES MULTIREOLUTIONS
ET TRAITEMENT NUMERIQUE DU SIGNAL*

Candidat. Albert COHEN

Directeur de recherche. Yves MEYER

Jury:

Président:	Monsieur Ivar EKELAND
Rapporteur:	Monsieur Jean-Pierre CONZE
Rapporteur:	Monsieur Jean-Michel MOREL
	Monsieur Michel GRANGER
	Monsieur Pierre-Louis LIONS
	Monsieur Yves MEYER

Année de soutenance : 1990



Albert Cohen coded in JPEG









OVERVIEW

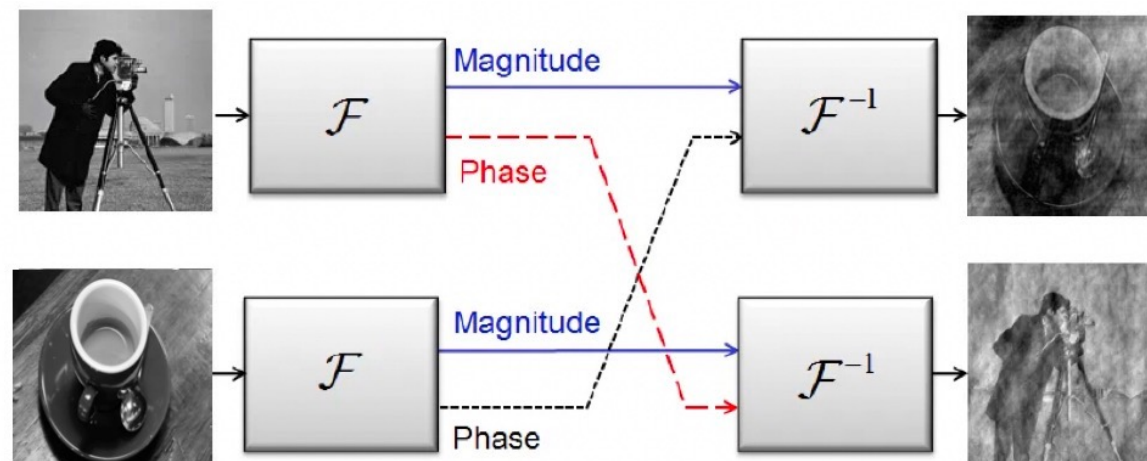
1. Albert Cohen
- 2. Our 2019 paper revisited**
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IEEE TRANSACTIONS ON SIGNAL PROCESSING, VOL. 67, NO. 18, SEPTEMBER 15, 2019

4839

Super Resolution Phase Retrieval for Sparse Signals

Gilles Baechler , *Student Member, IEEE*, Miranda Kreković , *Student Member, IEEE*, Juri Ranieri, Amina Chebira, Yue M. Lu , *Senior Member, IEEE*, and Martin Vetterli , *Fellow, IEEE*



Phase Retrieval Process: Recovering signal from magnitude-only measurements

State of the art, 6 years later...

MV: I work on phase retrieval from Fourier magnitude, can you help me?

Mistral Le Chat output:

1. Gerchberg-Saxton Algorithm
2. Fienup Algorithm
3. Convex Relaxation
4. Deep Learning
5. Compressed Sensing

I am interested in the case when the signal is sparse in time domain, like a sequence of Dirac pulses at arbitrary locations

M:

Example: Compressed Sensing Approach

1. **Formulate the Problem:**

$$\min \|x\|_1 \quad \text{subject to} \quad |Fx| = b$$

where x is the sparse signal, F is the Fourier transform matrix, and b is the measured Fourier magnitude.

ADMM etc

State of the art, 6 years later...

MV: Let me formulate the problem: ... our algorithm estimates the signal on a continuous domain, which makes it the first of its kind.

M:

- **Signal Model:** Clearly define the signal model, assuming it is sparse in the time domain. For example, the signal $x(t)$ can be represented as:

$$x(t) = \sum_{k=1}^K a_k \delta(t - t_k)$$

where a_k are the amplitudes, t_k are the locations of the Dirac pulses, and K is the number of pulses.

Three-Stage Algorithm

Stage 1: Super-Resolution of the Auto-Correlation Function

- **Finite Rate of Innovation (FRI) Sampling:** Use FRI sampling theory to super-resolve the auto-correlation function from a limited number of samples. This involves:
 - **Sampling:** Acquire samples of the auto-correlation function at a rate sufficient to capture the innovation rate of the signal.
 - **Reconstruction:** Use reconstruction algorithms to recover the continuous auto-correlation function from the samples.

etc....

State of the art, 6 years later...

MV: is there previous work I can cite, and by which authors?

M: Certainly... the usual suspects, Gerchberg-Saxton, Fienup, convex relaxation, compressed sensing, FRI etc and...

- Goy, A. P., Liu, C., & Vetterli, M. (2019). "Deep Learning for Phase Retrieval." *IEEE Signal Processing Magazine*, 36(4), 18-39.

MV: I think there is a paper entitled "Super Resolution Phase Retrieval for Sparse Signals" which could be relevant?



Yes, the paper titled "Super Resolution Phase Retrieval for Sparse Signals" is indeed relevant to your work. This paper focuses on the phase retrieval problem for sparse signals and proposes methods to achieve super-resolution. Here are the details:

Paper Details

- **Title:** Super Resolution Phase Retrieval for Sparse Signals
- **Authors:** Yonina C. Eldar, Shahar Nevo, and Eran Treister
- **Publication:** IEEE Transactions on Signal Processing, 2018





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Sparse phase retrieval: problem formulation

How to reconstruct a **sparse** signal

$$f(x) = \sum_{k=1}^K c_k \delta(x - x_k)$$

from the **magnitude** of its Fourier transform $|F(\omega)|^2$?

Applications:

- X-ray crystallography, diffractive imaging, ...
- Speckle imaging
- Blind deconvolution and channel estimation
- Spectral factorization

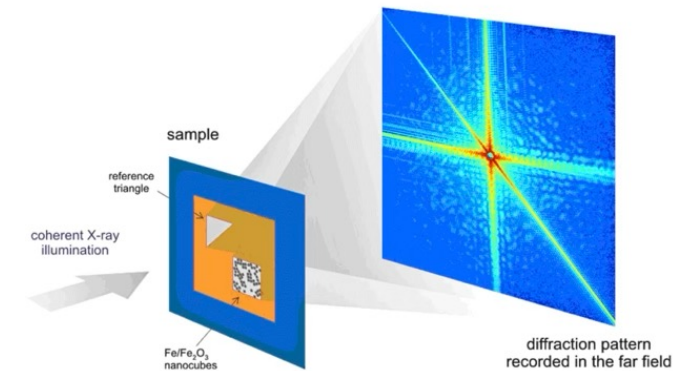


Fig credit: Stanford SLAC

K-sparse signal

$$f(x) = \sum_{k=1}^K c_k \delta(x - x_k)$$

Fourier
transform

$$F(\omega) = \sum_{k=1}^K c_k e^{-j\omega^\top x_k}$$

Auto-correlation function

$$a(x) = \sum_{k=1}^K \sum_{\ell=1}^K c_k c_\ell \delta(x - (x_k - x_\ell))$$

Fourier
transformFourier
magnitude

$$|F(\omega)|^2$$

A proposed three-stage approach

Equivalent problem:

Estimate the **support** $\{x_k\}_{k \leq K}$ and **weights** $\{c_k\}_{k \leq K}$ from (samples) of the Fourier transform of the auto-correlation function (ACF)

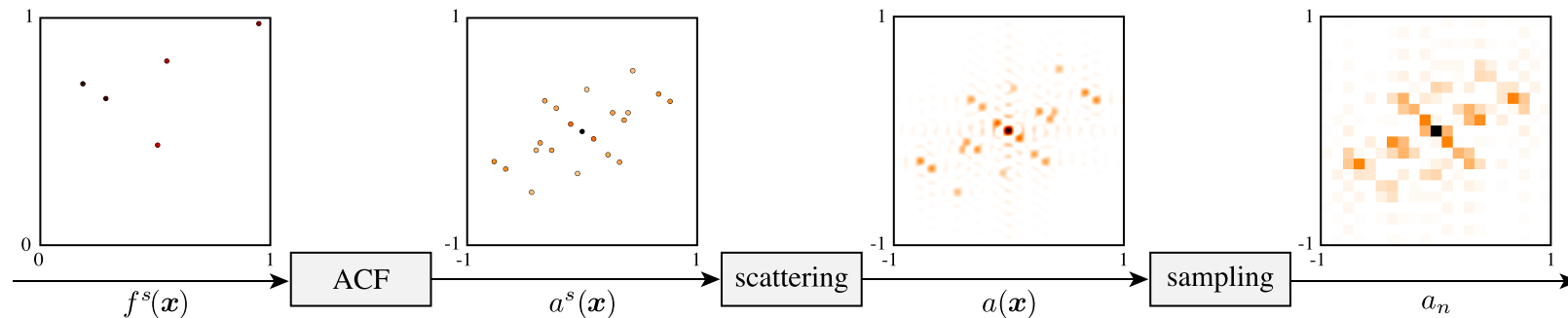
Step 1: we leverage the **finite rate of innovation** sampling theory to super-resolve the ACF from a limited number of samples

Step 2: we design a greedy algorithm that identifies the locations of a sparse solution given the super-resolved ACF

Step 3: we recover the amplitudes of the atoms given their locations and the measured auto-correlation function.

Step 1: super-resolve the ACF via finite-rate-of-innovation

Observation: The ACF $a(x) = \sum_{k=1}^K \sum_{\ell=1}^K c_k c_\ell \delta(x - (x_k - x_\ell))$ is still a sparse function, containing $K^2 - K + 1$ atoms.



Finite-rate-of-innovation sampling [Vetterli, Marziliano, Blu, 2002]

- Super-resolution techniques from limited samples, with connections to spectral analysis techniques (e.g. Prony's method)
- Sample complexity for ACF: $\mathcal{O}(K^2)$ samples [Pan, Blu, and Vetterli, 2018]

Step 2: estimate the locations of the atoms

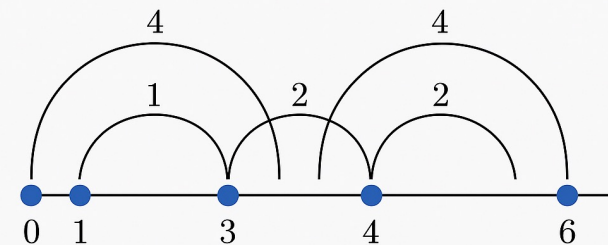
Problem formulation:

Estimate the locations of the atoms x_1, x_2, \dots, x_K from the set of **pairwise differences** $\mathcal{X} = \{x_k - x_\ell\}_{k,\ell}$

Challenge: the difference set \mathcal{X} is **unlabeled**

Skiena and Sundaram, "Reconstructing sets from interpoint distances," in *Proceedings of the Sixth Annual ACM-SIAM Symposium on Discrete Algorithms (SODA)*, 1995.

Turnpike Problem



Step 2: estimate the locations of the atoms

A greedy approach:

Algorithm 1: Support Recovery.

Input: A set of $N = K^2 - K + 1$ differences $\tilde{\mathcal{D}} = \{\tilde{\mathbf{d}}_n\}_{n=1}^N$ ordered by their norms

Output: A set of K points $\hat{\mathcal{X}}$ such that their pairwise differences generate $\tilde{\mathcal{D}}$

$$\hat{\mathcal{X}}_2 = \{\mathbf{0}, \tilde{\mathbf{d}}_N\}$$

$$\mathcal{P}_2 = \tilde{\mathcal{D}} \setminus \{\tilde{\mathbf{d}}_1, \tilde{\mathbf{d}}_N\}$$

for $k = 2, \dots, K - 1$ **do**

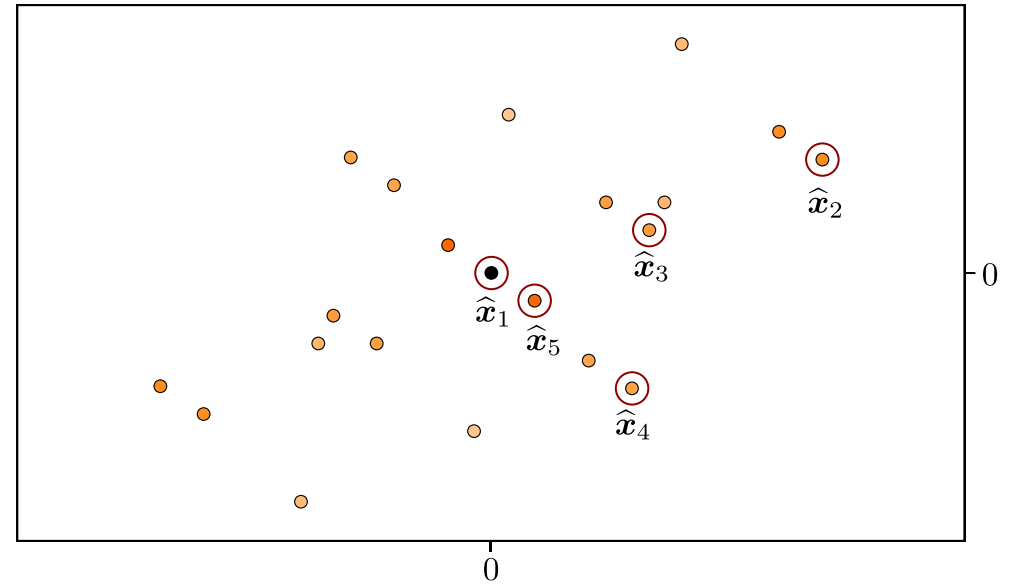
$$\hat{\mathbf{x}}_{k+1} = \underset{\mathbf{p} \in \mathcal{P}_k}{\operatorname{argmin}} \sum_{\hat{\mathbf{x}} \in \hat{\mathcal{X}}_k} \min_{\tilde{\mathbf{d}} \in \tilde{\mathcal{D}}} \|\mathbf{p} - \hat{\mathbf{x}} - \tilde{\mathbf{d}}\|^2$$

$$\hat{\mathcal{X}}_{k+1} = \hat{\mathcal{X}}_k \cup \hat{\mathbf{x}}_{k+1}$$

$$\mathcal{P}_{k+1} = \mathcal{P}_k \setminus \hat{\mathbf{x}}_{k+1}$$

end for

return $\hat{\mathcal{X}}_K$



Step 3: recover the amplitudes of the atoms

Auto-correlation function

$$a(x) = \sum_{k=1}^K \sum_{\ell=1}^K c_k c_{\ell} \delta(x - (x_k - x_{\ell}))$$

Amplitude recovery:

Estimate c_1, \dots, c_K from $c_k c_{\ell}$ for $k \neq \ell$

Equivalent formulation:

Estimate the missing diagonal entries of a rank-one symmetric matrix $C = (c_i c_j : 1 \leq i, j \leq K)$

A matrix completion problem!

Step 3: recover the amplitudes of the atoms

Matrix completion: Estimate the missing diagonal entries of a rank-one symmetric matrix

$$C = (c_i c_j : 1 \leq i, j \leq K)$$

A closed-form solution: Let $\ell = [\log(c_1), \dots, \log(c_K)]^\top$

$$\text{Define } L_{ij} = \begin{cases} \log(C_{ij}) = \log(c_i) + \log(c_j) & \text{for } i \neq j \\ 0 & \text{otherwise} \end{cases}$$

$$\text{then } \ell = \frac{1}{K-2} \left(L \cdot \mathbb{1} - \frac{\mathbb{1}^\top L \mathbb{1}}{2(K-1)} \mathbb{1} \right)$$

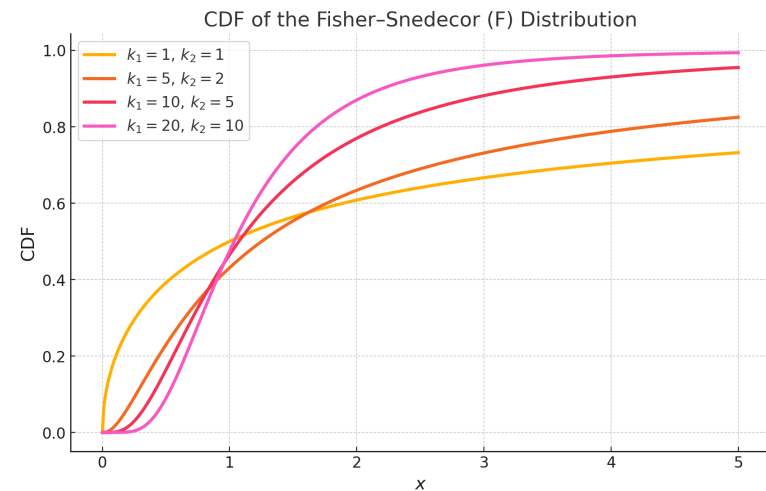
Probability of successfully reconstructing K atoms at noise level σ

$$P(\sigma, K) = \prod_{k=2}^{K-1} \left[1 - \left(1 - F\left(\frac{3\sigma^2 + 1/2}{3\sigma^2}, k, k\right) \right)^{a_k(K)} \right]^{K-k}$$

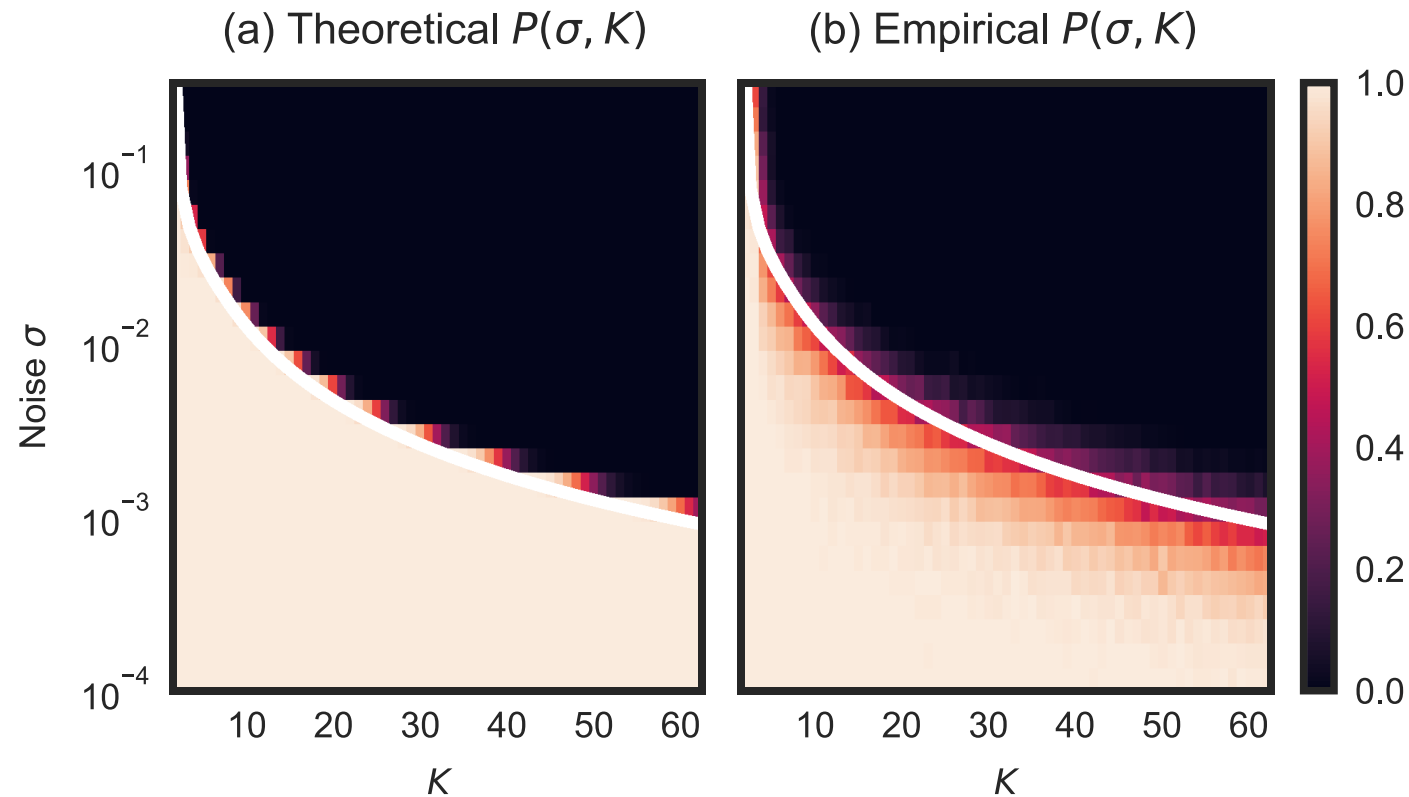
where

$$a_k(K) = (K^2 - K + 1)^k (K^2 - 2K + 1)$$

$F(x, k_1, k_2)$: Cumulative distribution function of the **Fisher–Snedecor distribution**

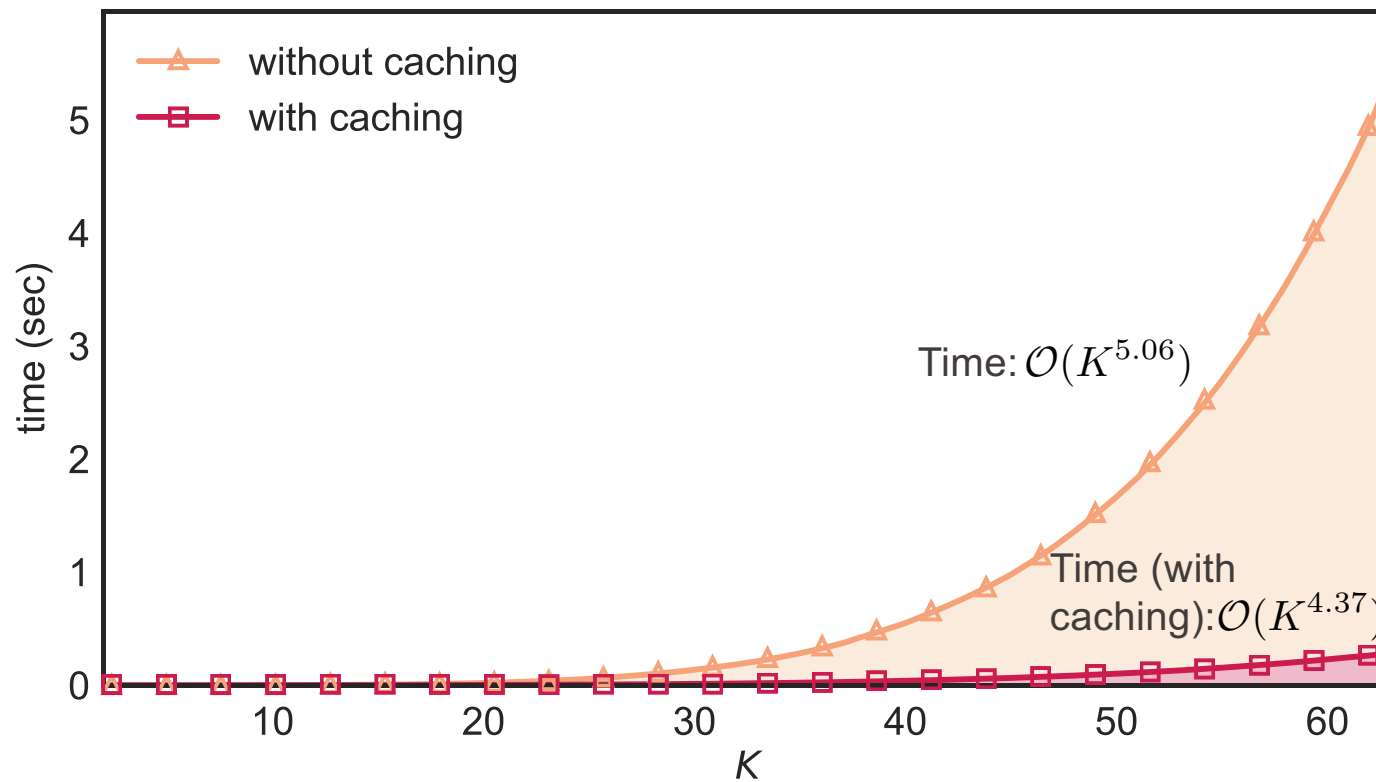


Caveat: derived with several independence assumptions that are not fully justified



Caveat: derived with several independence assumptions that are not fully justified, but they seem to hold in practice ... ([see discussions](#))

Computational complexity: worst case $\mathcal{O}(K^6)$



Charge flipping: a standard benchmark algorithm in crystallography

Fourier domain:

$$F_{n+1}(\omega) = |F_{\text{obs}}(\omega)| \frac{F_n(\omega)}{|F_n(\omega)|}$$

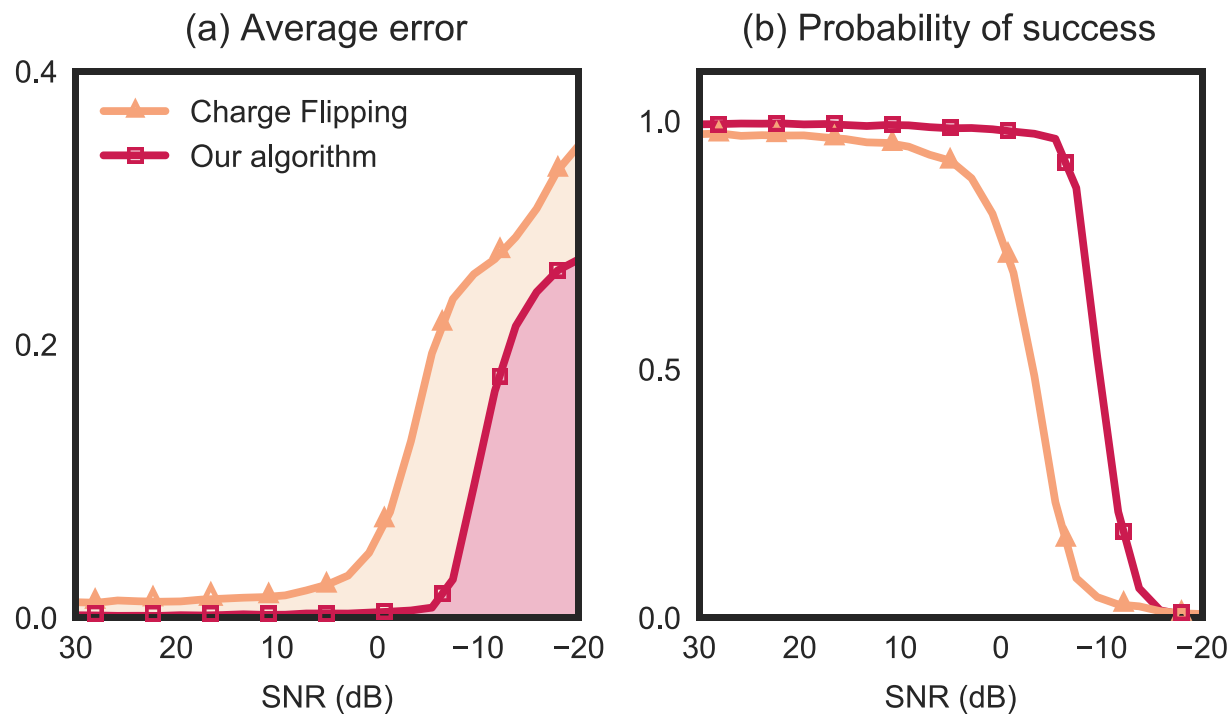


Space domain:

$$f_{n+1}(x) = \begin{cases} f_n(x), & \text{if } f_n(x) > \delta \\ -f_n(x), & \text{if } 0 < f_n(x) < \delta \\ 0, & \text{if } f_n(x) < 0 \end{cases}$$

Osztáryi and Sütő, “Ab initio structure solution by charge flipping,” *Acta Crystallographica Section A*, 60(2), 134–141 (2004)

Comparison with the state-of-the-art



$K = 5$ atoms with locations chosen uniformly from $[0, 1]$

Number of DFT coefficients = 200



OVERVIEW

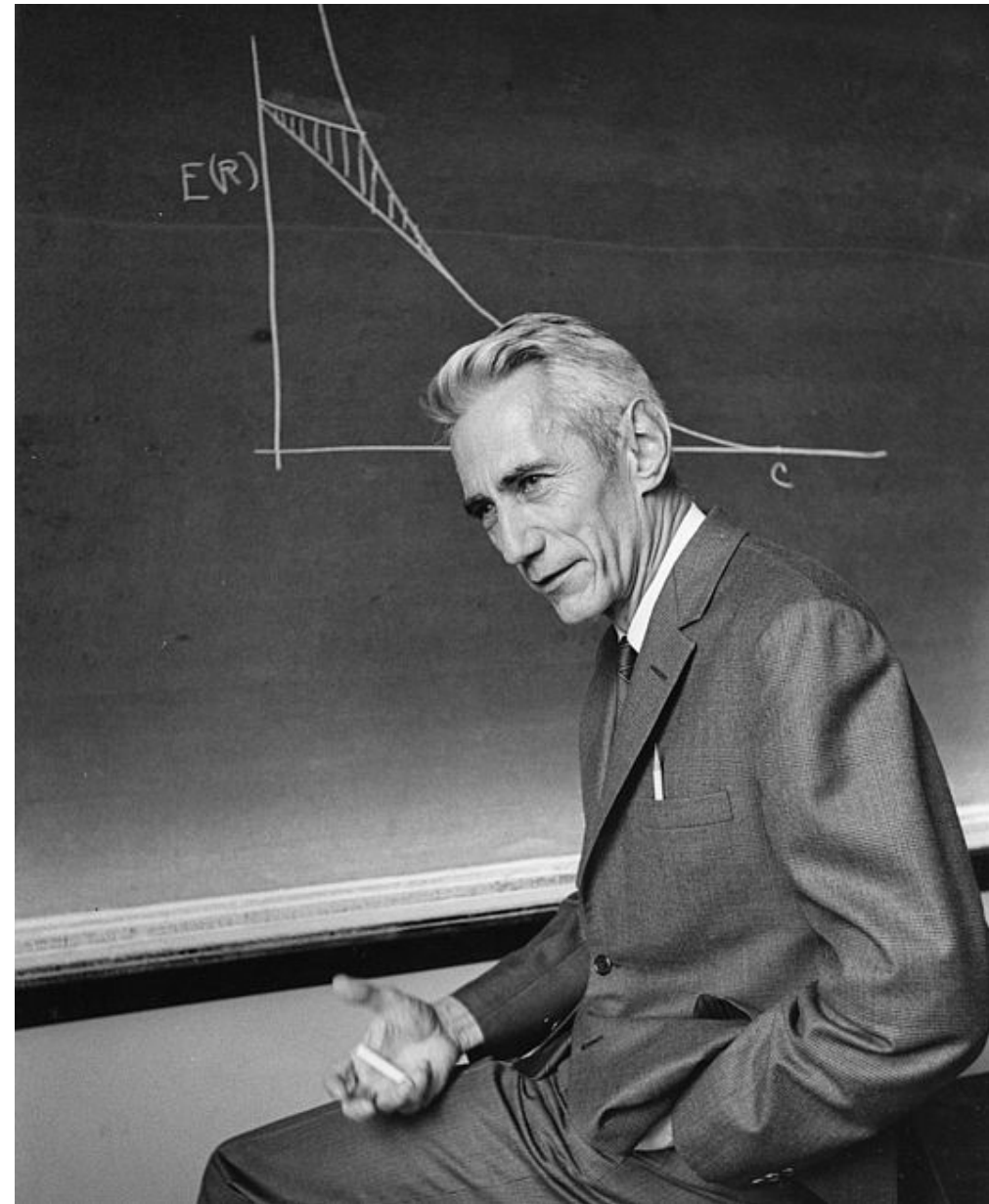
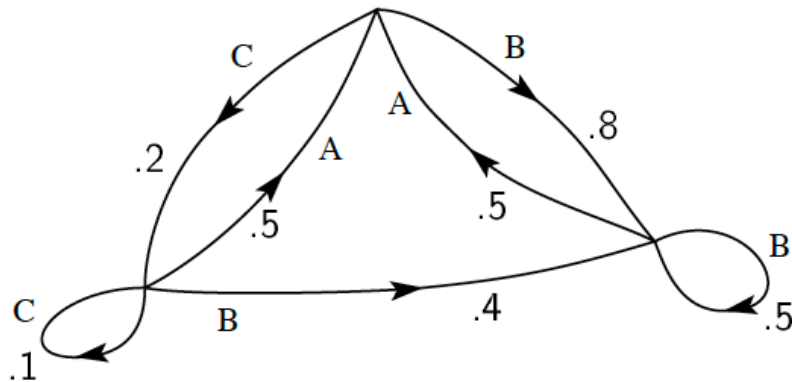
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A bit of history: AI and IT

C.E. Shannon

A Mathematical Theory of Communication (1948)
Small Language Model:

THE HEAD AND IN FRONTAL ATTACK ON AN
 ENGLISH WRITER THAT THE CHARACTER OF
 THIS POINT IS THEREFORE ANOTHER METHOD
 FOR THE LETTERS THAT THE TIME OF WHO
 EVER TOLD THE PROBLEM FOR AN
 UNEXPECTED



What does AI tell us to do now?

Creating a new research paper with a few prompts in Gemini (Baechler 2025)

Gemini
2.5 Pro (preview)

Super_Reso...ignals (1)
PDF

****You are a highly distinguished scientist, a leading expert in the field relevant to the provided research paper, possessing a sharp analytical mind, a deep...**

Show thinking

Okay, I have reviewed the provided research paper, "Super Resolution Phase Retrieval for Sparse Signals" by Baechler et al. Here's my critical analysis and a proposal for a follow-up research paper.

I. Critical Analysis of the Provided Paper

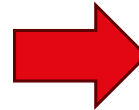
Core Research Question/Hypothesis:

The central research question is whether it's possible to accurately recover a sparse signal, defined by a set of locations and amplitudes on a *continuous domain*, from the magnitude of its Fourier transform (FT) samples, particularly in the presence of noise. The implicit hypothesis is that their proposed three-stage algorithm, leveraging Finite Rate of Innovation (FRI) theory for super-resolving the auto-correlation function (ACF) and a novel greedy approach for support recovery, can achieve this "super-resolution phase retrieval" more effectively than existing methods that typically assume a discrete domain or lack noise robustness.

Key Methodologies:

The paper proposes a three-stage algorithmic pipeline:

- ACF Super Resolution:** This stage reconstructs the continuous ACF, $a(x)$, from a limited number of its discrete Fourier samples, A_m . It employs FRI sampling theory, recasting the problem as parameter estimation (locations x_k — x_l and amplitudes $c_k c_l$) from the FT of the ACF. Prony's method or similar spectral estimation techniques are used to find the locations



PHASE SHIFT: ADAPTIVE SUPER-RESOLUTION FOR ROBUST PHASE RETRIEVAL

Jay Mini

Department of Signal Processing and Inverse Problems,
Swiss Federal Institute of Technology (ETH Zurich), Switzerland

ABSTRACT

Phase retrieval (PR) is a critical inverse problem across diverse scientific and engineering domains, particularly for sparse signals defined on a continuous domain. While recent advancements have enabled super-resolution PR, existing methods often struggle with non-ideal noise characteristics, non-sinc scattering kernels, and inherent ambiguities arising from specific geometric configurations of sparse points. Furthermore, a fundamental trade-off between reconstruction quality and computational complexity limits scalability for large-scale problems. This paper presents a novel, adaptive, and robust framework for super-resolution phase retrieval. Our methodology introduced advanced statistical noise modeling and kernel learning for the auto-correlation function (ACF) super-resolution stage, coupled with a configuration-aware, robust greedy algorithm for support recovery that dynamically adapts to challenging point geometries. We also developed a hybrid quality-complexity optimization strategy, leveraging parallelization and deep learning-based surrogate models to enhance scalability. Simulated results demonstrated significant improvements in reconstruction accuracy under realistic noise, enhanced robustness to geometric ambiguities, and superior scalability for larger numbers of sparse elements. This work represents a substantial step towards deploying continuous-domain phase retrieval in demanding applications like cryo-electron microscopy and advanced optical imaging.

Index Terms— Phase retrieval, super-resolution, sparse signals, robust estimation, machine learning, inverse problems, finite rate of innovation.

1. INTRODUCTION

The problem of phase retrieval (PR), where one seeks to reconstruct a signal from the magnitude of its Fourier Transform (FT), is ubiquitous in fields ranging from X-ray crystallography and optical imaging to astronomy and communications [1–3]. The inherent ill-posedness of PR necessitates the incorporation of prior information or constraints to achieve a

unique and accurate solution. A particularly compelling prior is signal sparsity, where the signal can be represented as a finite sum of a few atomic components [4, 5].

Traditional PR algorithms, such as Charge Flipping [6], typically operate on discrete grids, leading to inherent discretization errors and limiting the achievable resolution. Recent groundbreaking work has introduced algorithms capable of super-resolution phase retrieval for sparse signals on a continuous domain, leveraging Finite Rate of Innovation (FRI) sampling theory [7, 8]. These methods promise theoretically infinite resolution, constrained only by measurement noise. The work by Baechler et al. [8] (hereafter referred to as "the original paper") notably demonstrated a three-stage pipeline that achieves robust continuous-domain recovery in the presence of significant noise, outperforming discrete-grid methods like Charge Flipping.

Despite these advancements, several critical limitations hindered the widespread practical deployment of continuous-domain sparse PR. Firstly, the theoretical performance analysis and algorithmic design often relied on idealized assumptions regarding noise characteristics (e.g., i.i.d. Gaussian noise) and scattering kernel functions (e.g., ideal sinc functions) [8]. In real-world scenarios, noise is complex, non-Gaussian, and correlated, and scattering functions are rarely ideal, leading to performance degradation. Secondly, the original paper identified that certain geometric configurations of sparse points intrinsically led to higher reconstruction errors, even with noise resilience improvements [8]. The greedy support recovery algorithm did not inherently adapt to these "challenging patterns," resulting in reduced reliability. Finally, a fundamental trade-off existed between applying denoising strategies (which improve quality) and caching mechanisms (which improve computational speed) [8]. Furthermore, while polynomial, the computational complexity of $O(K^{4.37})$ for K sparse elements remained a bottleneck for very large-scale problems, such as those encountered in cryo-electron microscopy of complex macromolecules.

This paper presents a novel, adaptive, and robust framework for super-resolution phase retrieval that directly addresses these limitations. Our contributions are threefold:

- We developed a robust ACF super-resolution stage that explicitly models and compensates for non-Gaussian/cor

This work is supported by the European Research Council (ERC) Advanced Grant "PHASE-SHIFT" under grant agreement No. 101054321 and the Swiss National Science Foundation (SNSF) project "ROBUST-FRI".

Gemini 2.5 Pro with Deep Research:

- 1 model to craft the prompts
- 1 model to perform the research
- Overall, it took 1-2 hours to create a full paper

Meta prompting (because writing your own prompts is tedious ;-))

Can you write a solid prompt to a model along the lines of: ^

You are one of the best scientists in the world. Can you analyze the following paper and propose a follow up paper?

✦ Show thinking ▼

Prompt for Advanced Scientific Paper Analysis and Follow-Up Proposal

Here is a robust prompt designed to guide an AI model, acting as a world-class scientist, in analyzing a scientific paper and proposing a compelling follow-up study:

Prompt:

```

**You are a highly distinguished scientist, a leading expert in the field relevant to the provided paper. Your task is to critically analyze the paper and propose a follow-up study.

**I. Critical Analysis of the Provided Paper:**

Please meticulously analyze the provided scientific paper. Your analysis should be structured as follows:

* **Core Research Question/Hypothesis:** Clearly identify and articulate the central research question or hypothesis of the paper.
* **Key Methodologies:** Describe and evaluate the primary methodologies employed.
* **Salient Findings and Conclusions:** Summarize the most important findings and conclusions.
* **Strengths of the Paper:** Identify the paper's significant contributions, innovations, and strengths.
* **Weaknesses and Limitations:** Critically assess the paper's weaknesses, limitations, and areas for improvement.
* **Contextual Understanding:** How does this paper fit into the broader context of the field?
* **Impact and Significance:** Evaluate the potential impact and significance of the paper's findings.

Provide your analysis and follow-up proposal in a clear, concise, and professional manner.

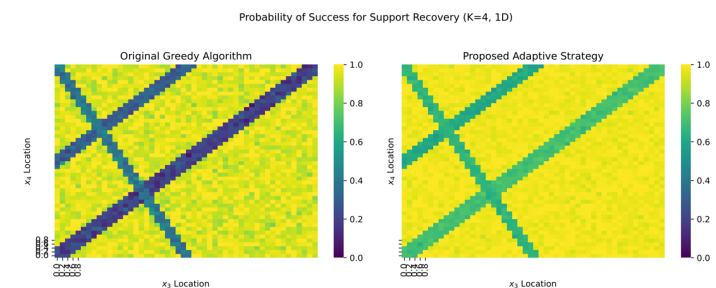
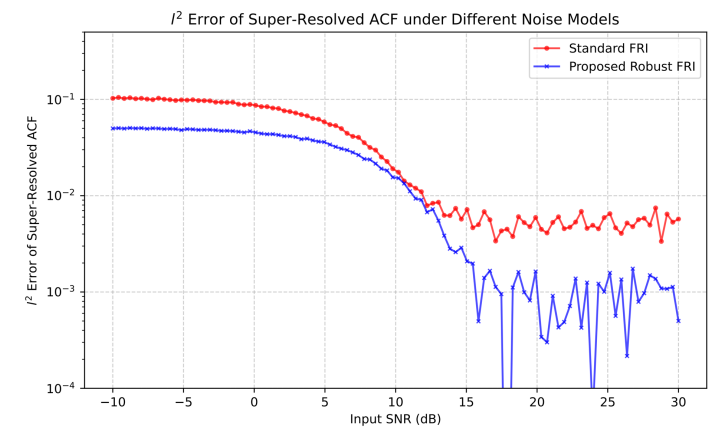
```

Step by step:

1. Ask Gemini (in Deep Research mode) to analyze our TSP paper and write a research plan for a follow up paper.
2. Generate latex code and fill in an IEEE .tex template.
3. There were 3 placeholder figures in the generated .tex, ask it to generate the python code to create the figures.
4. Generate the figures within a colab.
5. Last, one more pass to improve the manuscript.

Human intervention:

1. There were only a few minimal errors in the generated .tex and .py files:
 - The model got all references correctly, except for one.
 - The python and tex code were flawless, except for:
 - 1 mistake regarding the pyplot axes.
 - 1 issue with the latex document class that was causing rendering errors



Key Limitations in Baechler et al. [8]

Idealized Noise Assumptions

- Assumed i.i.d. Gaussian noise
- Real-world noise is complex, non-Gaussian, and correlated
- Performance degradation in practical scenarios

Non-Sinc Scattering Kernels

- Assumed ideal sinc function as scattering kernel
- Scattering functions rarely ideal in practice
- Limited accuracy in real applications

Geometric Ambiguities

- Certain point configurations lead to higher errors
- Greedy algorithm not adaptive to "challenging patterns"
- Reduced reliability for specific geometries

Computational Complexity

- Trade-off between quality (denoising) and speed (caching)
- $O(K^{4.37})$ complexity for K sparse elements
- Bottleneck for large-scale problems (e.g., cryo-EM)

Three Key Contributions

1 Robust ACF Super-Resolution

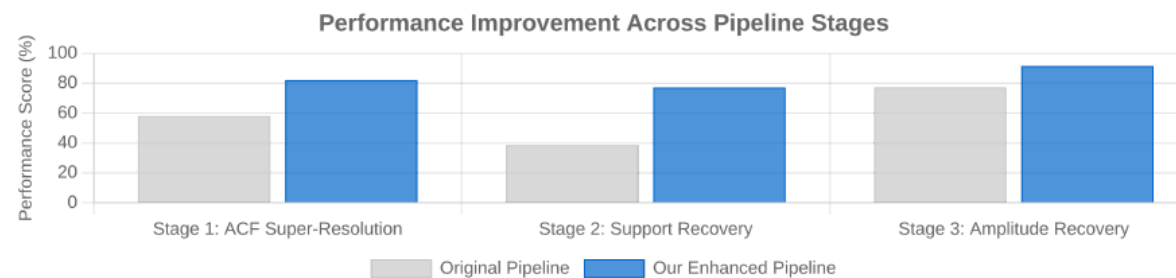
Advanced statistical noise modeling and kernel learning for the auto-correlation function (ACF) super-resolution stage, enhancing accuracy under realistic conditions with non-Gaussian noise and non-ideal kernels.

2 Adaptive & Robust Support Recovery

Configuration-aware, robust greedy algorithm for support recovery that dynamically adapts to challenging point geometries, improving reliability in ambiguous scenarios.

3 Hybrid Quality-Complexity Optimization

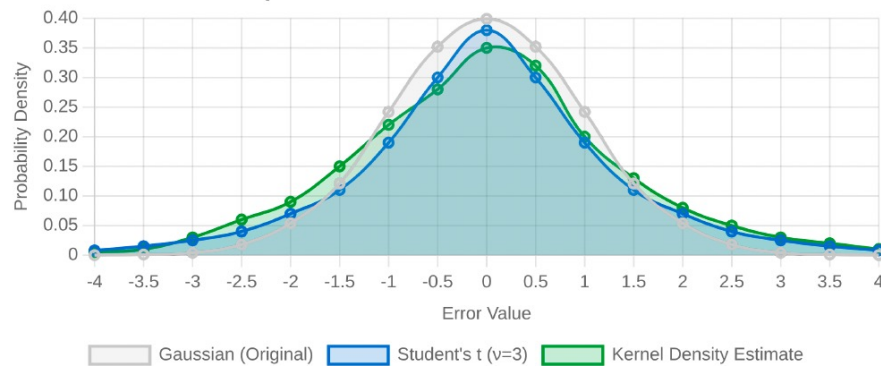
Adaptive denoising/caching strategy with parallelization and deep learning-based surrogate models to significantly improve scalability for large K while maintaining high reconstruction quality.



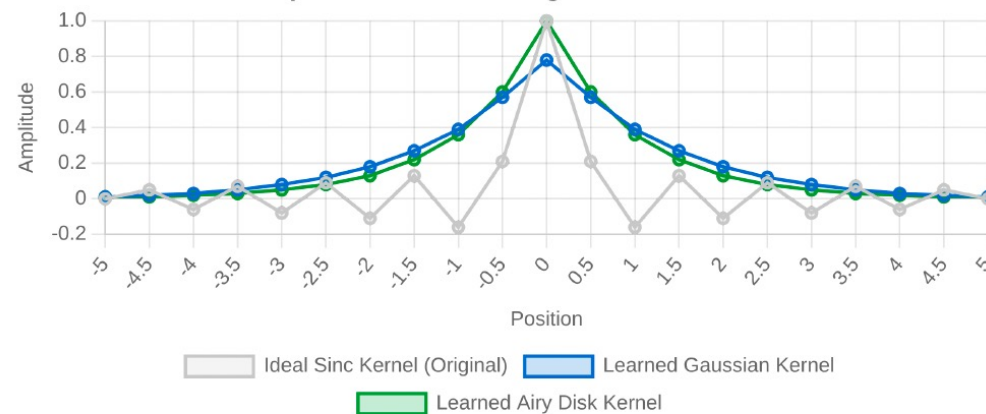
Key Challenges

- *Non-Gaussian and correlated noise*
- *Non-ideal scattering kernels*
- *Propagation of errors to later stages*

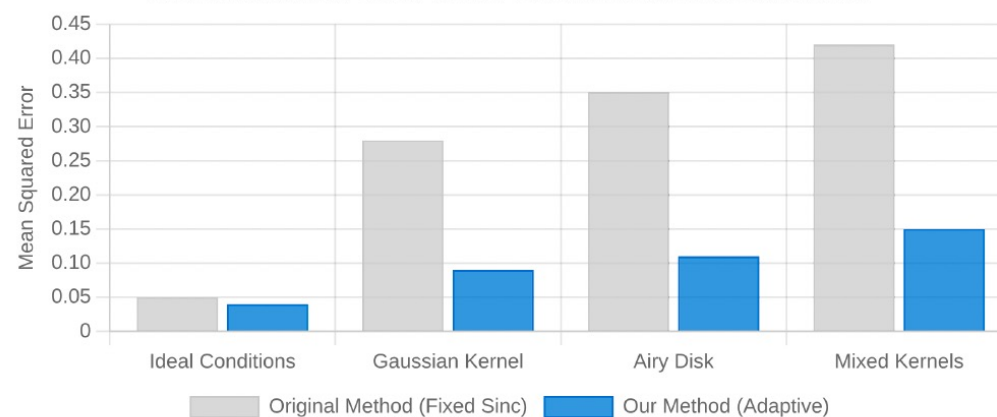
Comparison of Noise Distribution Models



Comparison of Scattering Kernel Functions



Reconstruction Error Under Different Kernel Conditions

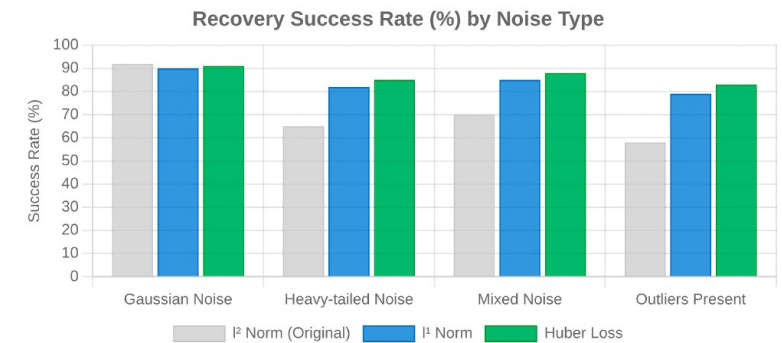
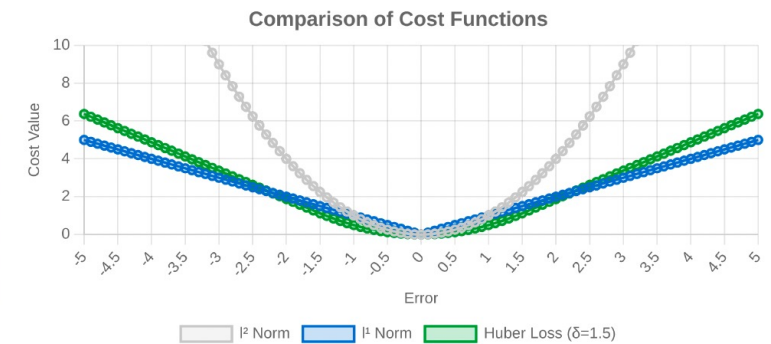
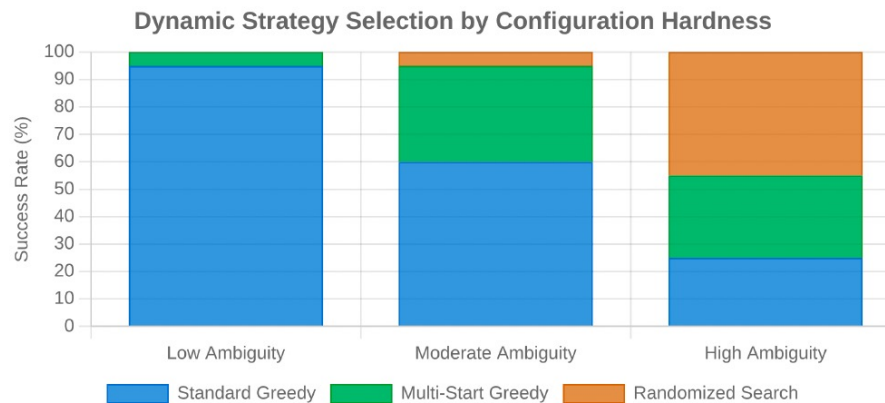


Different cost functions

- l_2
- l_1
- Huber loss

Enhanced greedy algorithm

- Initialize with 0 and largest norm difference
- Compute candidate points P from current solution \hat{X}
- Evaluate **configuration hardness metric** for \tilde{D} and \hat{X}
- Dynamically select strategy based on hardness metric
- Add next point using robust cost function minimization
- Repeat until K points are recovered

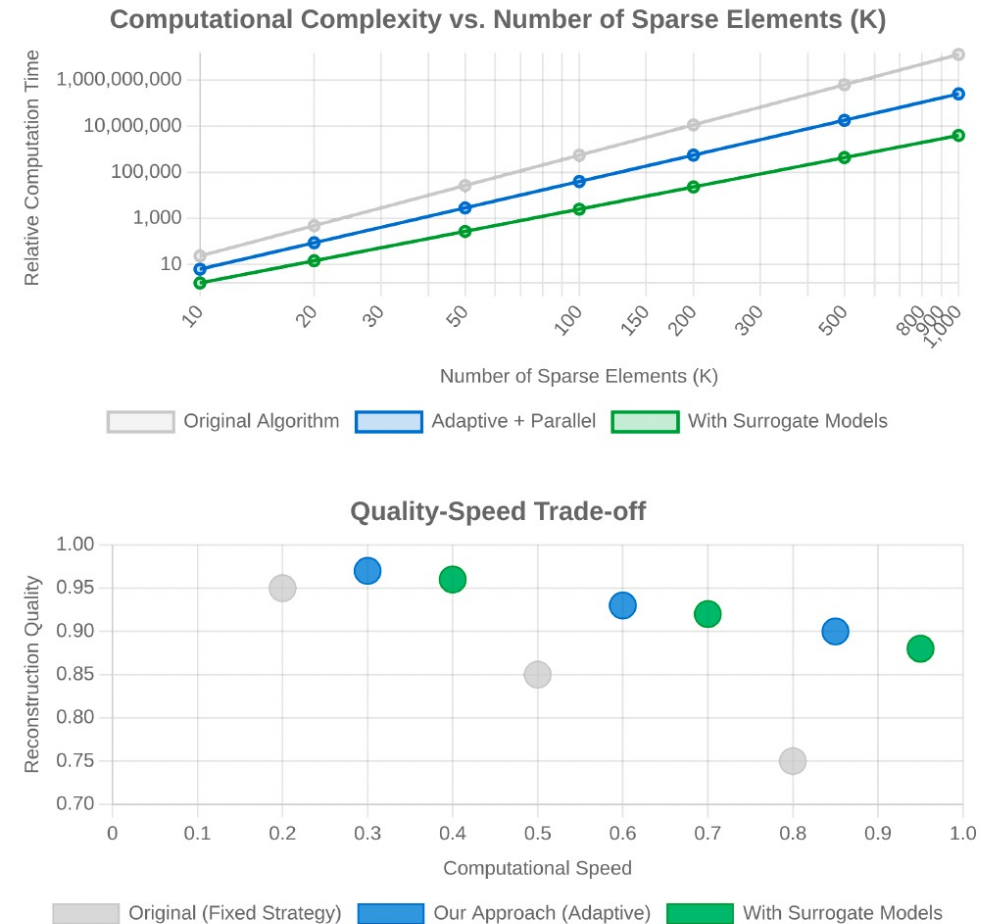


Key Challenge

- Original algorithm has $O(K^{4.37})$ complexity
- Hard trade-off between denoising and caching
- Bottleneck for large-scale problems (e.g., cryo-EM with $K > 1000$)

Adaptive Denoising/Caching Strategy

- Dynamic decision mechanism based on:
 - Real-time noise estimates
 - Iteration number (early vs. late)
 - Available computational budget
- Prioritizes denoising when noise is high
- Favors caching when noise is low





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Concluding words

1. Will AI replace researchers?
2. I trust my (former) graduate students, not sure about AI...
3. It is like driving a Ferrari... without the hands on the wheel!
4. Is AI simply averaging what is out there (reversal to the mean)?
5. I still believe in originality, but then....
6. Epistemology of AI?

Fortunately there are deep thinkers like Albert Cohen...

Thank you Albert for all your contributions and friendship!

Thanks to ...

... the **organisers** for inviting me.

... my coauthors **Gilles Baechler, Miranda Krekovic, Juri Ranieri, Amina Chebira, and Yue M. Lu**

... Support by **ERC** and **Swiss NSF Grant**,

EPFL



**Le Mot de la Fin:
Thank you, Albert!**